**A Full-scale Experimental Study on Simultaneous Energy Harvesting and Vibration Control of Bridge Stay Cables using Electromagnetic Inertial Mass Dampers**

Wenai Shen, Yuhang Hu, Yamin Li, Hongping Zhu\*, Songye Zhu

1. School of Civil Engineering and Mechanics, Huazhong University of Science and Technology, Wuhan 430074, China.

2. Department of Civil and Environmental Engineering, The Hong Kong Polytechnic University, Kowloon, Hong Kong

\*Corresponding author: [hpzhu@hust.edu.cn](mailto:hpzhu@hust.edu.cn)

两倍行距，新罗马字体，图和表都放在参考文献之后。

**Abstract**

**Key words**: stay cable, electromagnetic, inertial mass, circuit, energy harvesting, vibration mitigation

**1. Introduction**

1. **Description of Cable-EIMD System**

## **Configuration**

Figure 1. Configuration of cable-EIMD system for vibration control and energy harvesting

Figure 2. Configuration of prototype EIMD

## **Vibration Damping**

## **Power Flow and Efficiency**

## **3. Experimental Setup**

## **3.1 Test cable**

Figure 3. Schematic of full-scale cable vibration test setup

(a) Cable

(b) EIMD

(c) Energy harvesting circuit

(d) Data acquisition system

Figure 4. Pictures of full-scale cable vibration test setup

Table 1. Main parameters of full-scale stay cable and EIMD

## **3.2 Prototype EIMD**

## **3.3 Test Circuits**

## **3.4 Instrumentation**

## **3.5 Test Scenarios**

Table 2. Scenarios of full-scale stay cable vibration testing

## **4. Results**

## **4.1 Vibration Control Performance**

4.1.1 Damping characteristics

(a) Damping force-displacement

(b) Damping force-velocity

Figure 5. Measured damping force–displacement plot and damping force–velocity plot of the EIMD subjected to sine excitation (x Hz)

4.1.2 Free vibration response control

C:\Users\hyh\AppData\Local\Microsoft\Windows\INetCache\Content.Word\disp.eps

###### (a) Displacement

C:\Users\hyh\AppData\Local\Microsoft\Windows\INetCache\Content.Word\acc.eps

(b) Acceleration

Figure 6. Free vibration responses of test cable with and without EIMD

Figure 6 illustrates the time histories of the free vibration displacement and acceleration at mid-span of test cable with and without EIMD (4ton, 2.48%, R=30Ω) respectively. It can be seen that the free vibration of test cable is effectively suppressed by EIMD, in which the displacement response is reduced from 80.89mm to 19.35mm with a rate of 76.08%, while the acceleration response is reduced from 4.252m/s2 to 2.034m/s2 with a rate of 52.16% simultaneously. In addition, the natural frequency of controlled cable with EIMD has slightly increased.



Figure 7. FFT spectra of free vibration responses of test cable with and without EIMD

Figure 7 shows the FFT spectra of the free vibration displacement at mid-span of test cable with and without EIMD (4ton, 2.48%, R=30Ω) respectively. A considerable reduction of peak value in the FFT spectra (from 60.60mm to 31.18mm) due to the installation of EIMD is observed. Another phenomenon is the natural frequency shift from 1.0605Hz to 1.0986Hz. This phenomenon can be explained by…

Table 3. Modal damping ratios of test cable with and without EIMD

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Test | | Mass/ton | | Position/% | Damping ratio  (1T) | Damping ratio  (5T) | Damping ratio  (10T) |
| Without control | - | | - | | 0.0049 | 0.0010 | 0.0002 |
| With control | | 0 | | 2.48 | 0.0295 | 0.0225 | 0.0190 |
| 5 | 0.0131 | 0.0188 | 0.0158 |
| 2 | | 2.48 | 0.0515 | 0.0266 | 0.0187 |
| 5 | 0.0348 | 0.0303 | 0.0199 |
| 4 | | 2.48 | 0.0384 | 0.0366 | 0.0256 |
| 5 | 0.0556 | 0.0260 | 0.0248 |
| 6 | | 2.48 | 0.0537 | 0.0381 | 0.0240 |
| 5 | 0.0859 | 0.0287 | 0.0179 |
| 8 | | 2.48 | 0.0885 | 0.0392 | 0.0248 |
| 5 | 0.0309 | 0.0286 | 0.0233 |
| 10 | | 2.48 | 0.0256 | 0.0305 | 0.0273 |
| 5 | 0.0261 | 0.0189 | 0.0163 |

As aforementioned, a series of experiments employing EIMDs of different inertial masses (0, 2, 4, 6, 8, 10ton) and installation positions (2.48%, 5% of test cable) are conducted. The average damping ratios of test cable with and without EIMD, identified by the vibration attenuation method with the computing interval of 1, 5 and 10 vibration periods respectively, are presented in table 3. We observed that the damping ratio of the uncontrolled cable is quite low. With the aid of EIMD, the damping ratio was enhanced to……

4.1.3 Forced vibration response control



Figure 7. Displacement time histories of test cable with and without EIMD subjected to sine sweep excitation (the first vibration mode, frequency range: 0.95–1.20 Hz)

The vibration control performance of EIMD is further more examined when the test cable is subjected to sine sweep excitation. Figure 7 shows the displacement response under sine sweep excitation at mid-span of test cable with and without EIMD (4ton, 5%, R=50Ω). The frequency of sine sweep excitation ranges from 0.95Hz to 1.20Hz with the sweeping speed of 0.001Hz/s, which corresponds to the first vibration mode of test cable. Comparing with the uncontrolled test cable, a significant reduction in the displacement response can be observed by employing EIMD, and the peak displacement response is suppressed from 43.51mm to 33.47mm. It is noted that the natural frequency slightly shifts from 1.085Hz to 1.146Hz after the installation of EIMD. (explain:……)

4.1.4 Frequency response function



Figure 8. Displacement time histories of test cable with and without EIMD subjected to harmonic sweeping excitation (frequency range 0–4 Hz, 0.01Hz/s)

Figure 8 presents the time histories of displacement at mid-span of test cable under harmonic sweeping excitation with and without EIMD (4ton, 5%, R=14Ω). The frequency range (0 to 4Hz) of the harmonic sweeping excitations covers the first three modes of test cable. As is shown in this figure, the targeted modal vibration (mode 1) of test cable is suppressed from 30.67mm to 26.29mm due to the use of EIMD, along with the resonant frequency shift in mode 1. However, it is observed that the EIMD has an undesirable effect in response control for mode 2 and 3 of test cable.



Figure 9. Measured FRF of displacement responses to exciting force

Figure 9 shows the FRFs of displacement at mid-span of test cable under harmonic sweeping excitation with and without EIMD (4ton, 5%, R=14Ω). By the installation of EIMD, the peak magnitudes of the FRF, which corresponds to the first three natural frequencies (1.06Hz, 2.11Hz and 3.17Hz), have been considerably reduced by 12.251dB, 7.518dB and 5.370dB respectively. Notably, two dominant peaks can be seen for the controlled cable with EIMD near the first natural frequency, as well as two dominant frequencies (1.057Hz and 1.135Hz).

Equations:



Theoretical 

Theoretical 

Theoretical 



## **4.2 Energy Harvesting Performance**

4.2.1 Results of Free Vibration Case

The output power and energy harvesting efficiency of EIMD are assessed when different load resistors (0Ω to 50Ω) are connected in the resistor circuit cases. A typical time history of the measured voltage across the load resistor during the free vibration of test cable with EIMD (6ton, 5%, R=3) is presented in figure 10.

C:\Users\hyh\AppData\Local\Microsoft\Windows\INetCache\Content.Word\TimehistoryofVoltage.emf

Figure 10. Typical time history of measured voltage across the load resistor (free vibration)



(a) Damper is located at 2.48% of test cable

C:\Users\hyh\AppData\Local\Microsoft\Windows\INetCache\Content.Word\Pout_5%.emf

(b) Damper is located at 5% of test cable

Figure 11. Output power versus load resistance (free vibration)

Figure 11 compares the theoretical and testing average output powers versus the load resistances () in different test cases. The theoretical values of the average output power are calculated according to equation (x), while the testing results are calculated by the measured voltage across the load resistor. Figure 11 shows a good agreement between the theoretical and testing results. The maximum output powers, as well as their corresponding load resistances in different test cases are listed in Table 3. It is observed that the experimental result of average output power can achieve the maximums of 1.6961W (M=0ton, R=3Ω) and 2.0973W (M=6ton, R=3Ω), when the damper is located at 2.48% and 5% of test cable respectively.



(a) Damper is located at 2.48% of test cable



(b) Damper is located at 5% of test cable

Figure 12. Energy harvesting efficiency versus load resistance (free vibration)

Similar observations can be obtained from the energy harvesting efficiency versus load resistance curves shown in figure 12. The theoretical values of the energy harvesting efficiencies are calculated according to equation (x), while the testing results are calculated by equation (x) in which the average input power is calculated by the measured force and displacement of EIMD. Figure 12 shows a good agreement between the theoretical and testing results. The energy harvesting efficiencies as well as their corresponding load resistances in different test cases are also listed in Table 3. It is observed that the experimental result of energy harvesting efficiency can achieve the maximums of 19.28% (M=8ton, R=5Ω) and 23.34% (M=4ton, R=8Ω), when the damper is located at 2.48% and 5% of test cable respectively.

Table 4. Optimal resistance for output power and efficiency of EIMD (free vibration)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Damper position / % | Inertial Mass / ton | Optimal Resistance / ohm | Efficiency / % | Optimal Resistance / ohm | Output Power / W |
| 2.48 | 0 | 3 | 13.66 | 3 | 1.6961 |
| 2 | 3 | 15.97 | 3 | 1.3832 |
| 4 | 6 | 14.02 | 6 | 1.1160 |
| 6 | 2 | 15.28 | 2 | 1.2106 |
| 8 | 5 | 19.28 | 5 | 0.9296 |
| 10 | 3 | 16.10 | 8 | 0.6508 |
| 5 | 0 | 3 | 22.26 | 3 | 1.5600 |
| 2 | 3 | 16.88 | 5 | 1.0659 |
| 4 | 8 | 23.34 | 3 | 1.5159 |
| 6 | 3 | 15.89 | 3 | 2.0973 |
| 8 | 3 | 21.15 | 3 | 1.1020 |
| 10 | 3 | 18.78 | 3 | 1.1556 |

Table 3 shows the maximum output power and maximum energy harvesting efficiency, as well as their corresponding load resistances, in different test cases. The maximum output power ranges from 0.6508W to 2.0973W, and the maximum energy harvesting efficiency ranges from 13.66% to 23.34%. The optimal load resistances corresponding to the maximum output power and energy harvesting efficiency are identical in most test cases, which range from 2Ω to 6Ω. Considering that the parasitic damping coefficient () varies in different test cases, the theoretical values of optimal load resistance are also different. Thus, the experimental results fit fairly well with the theoretical predictions obtained by using equation (x).

4.2.2 Results of Resonant Vibration Case



(a) Damper is located at 2.48% of test cable



(b) Damper is located at 5% of test cable

Figure 4. Output power versus load resistance (resonant vibration)



(a) Damper is located at 2.48% of test cable

C:\Users\hyh\AppData\Local\Microsoft\Windows\INetCache\Content.Word\Eta_5%_subplot4.emf

(b) Damper is located at 5% of test cable

Figure 2. Energy harvesting efficiency versus load resistance (resonant vibration)

Table 5. Optimal resistance for output power and efficiency of EIMD (resonant vibration)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Damper position / % | Inertial Mass / ton | Optimal Resistance / ohm | Efficiency / % | Optimal Resistance / ohm | Output Power / W |
| 2.48 | 0 | 1 | 1.27 | 1 | 0.0097 |
| 2 | 12 | 0.66 | 1 | 0.0215 |
| 4 | 2 | 2.99 | 1 | 0.4023 |
| 6 | 3 | 3.39 | 3 | 0.0869 |
| 5 | 0 | 3 | 3.73 | 1 | 0.2458 |
| 2 | 5 | 2.27 | 1 | 0.2781 |
| 4 | 3 | 4.20 | 1 | 0.7165 |
| 6 | 5 | 5.23 | 2 | 0.2237 |



Figure 3. Typical time history of measured voltage across the load resistor (the first vibration mode, frequency range 0.95–1.20 Hz)

备用图

TimehistoryofVoltage



C:\Users\hyh\AppData\Local\Microsoft\Windows\INetCache\Content.Word\TimehistoryofVoltage.emf



## **5. Discussions**

##### On the optimal damping coefficient for EIMD design

##### On the maximum achievable modal damping ratio

##### On the energy dissipation capability of EIMD

## **6. Conclusions**

## **Acknowledgement**

The authors are grateful for the financial support from the National Natural Science Foundation of China (Grant Nos.: 51508217 & 51178203).

## **Appendix**

Formulation of cable-EIMD system and its complex eigenvalue analysis

## **References**

# **Figures**

**Figure 1.** Configuration of EMDEH for stay cable vibration control and energy harvesting

# **Tables**

**Table 1.** Main parameters of test stay cable

|  |  |  |  |
| --- | --- | --- | --- |
| Item | Value | Item | Value |
| Mass per unit length, m | 0.442 kg/m | Diameter | 4 mm |
| Cable length, *l* | 5.85 m | Cross-sectional area | 7.28 mm2 |
| Inclination | 15.5° | Young’s modulus |  |
| Static tension force | 980 N | Axial stiffness |  |